1. Introduction

One of the oldest and still controversial issue in the economic analysis of taxation (and consequently of discretionary fiscal policy), both on theoretical and empirical grounds, is the appropriate mix of income and consumption taxes which maximizes welfare, and whether and how it affects economic growth. More recently, the issue of a revenue neutral shift from direct to indirect taxation has become a key-issue of the policy discussion. Especially in Europe, where the fiscal consolidation is likely to turn into pro-cyclical fiscal policies, and thus the neutral tax shift is viewed as a potential mean to maintain fiscal discipline while preserving long term growth, and, possibly, giving some impulse to short run aggregate demand.

In what follows I expose the main results of a research program aimed to assess, theoretically and empirically, the macroeconomic effects of switching the tax burden from productive inputs to consumption (see Felli et al., 2011). The analysis is limited to Italy, but we shall extend both coverage and methodology.

The theoretical reference framework is an endogenous growth model of the AK type with elastic labor supply (see Turnovsky 2000, for a reference). To explore the impact of tax composition on Italy economic performance, a dynamic stochastic structural macroeconomic model (in the Cowles Commission tradition) of the Italian economy is exploited. This model, called Merman (Felli and Gerli, 2002), is centered around a supply block where an endogenous TFP function of causal order zero is
formulated and estimated. In the estimated equation (displaying an error correction specification), a tax ratio variable, representing the tax structure in Italy, and expressed as the fraction of direct to indirect taxes, is comprised among the regressors (human capital, labor market rules, and other institutional and policy factors). The results of the simulations based on Italian official national accounts provide an argument for the reduction of the direct to indirect tax ratio. Indeed, differently from previous studies that find only a slight link between taxes and growth, our investigation reveals that even a revenue neutral switch from direct to indirect taxes is likely to generate efficiency gains, which lead to higher growth rates of per capita GDP. I argue that, in this special case of rerunning history simulations, the basic objection of the Lucas critique to structural econometrics does not apply, or at least its scope is of second order.

The channel through which a change in the tax composition affects economic growth can be described as follows. Ceteris paribus, that is keeping the government spending unchanged, a reduction in income tax (direct taxation), completely financed through an increase in consumption tax (indirect taxes), induces a net increase in labor supply – the lower wage tax increases work effort – and consequently in employment, which in turn raises the productivity of capital and implies a higher equilibrium growth rate. In other words, the positive effect on the growth rate of a lower wage tax dominates the opposite effect of a higher consumption tax. In the theoretical model (see Appendix), this result crucially depends on the relative size of two parameters: the fraction of time devoted (allocated) to leisure, \( l \), and the leisure ‘elasticity’ in the individual utility function, \( \psi \), which measures how much leisure affects the individual welfare. Provided that \( l \) is of a sizeable amount (as it happens in the real world), the income (wage) tax effect prevails, so that the proposed tax shift reduces leisure and stimulates growth. As it is shown in the Appendix (Proof 5), the inequality, ensuring that the effect of the income tax reduction dominates, must hold to avoid an explosive growth path, and in fact it is satisfied along the balanced growth path, given the transversality condition and the non-increasing returns to scale of the production function. Numerical solutions of the (theoretical) model confirm this result. Even Turnovsky (2000), in a model belonging to the same class of that illustrated in the Appendix, performs numerical solutions of his model and finds a strong support for the dominance of the income tax effect – he parametrizes \( l = 0.77 \), a plausible value given that the observed yearly fraction on total hours devoted to work is around \( \frac{2000}{8760} = 0.23 \) (hence \( l = 0.77 \)), so that \( \psi \) should
have an implausible high size (e.g. 3.4) in order to be at least equal to \( l \).

The paper is organized as follows. In the following section 2, I briefly review some results of the theoretical and empirical literature on the relationship between the tax structure and the economic performance. In section 3 the simulation exercise is outlined. Section 4 presents the main results and section 5 concludes. An appendix exposing the theoretical model follows.

2. Theoretical and Empirical Background

A broad class of endogenous growth models, either of the Lucas’ type (adding human capital to the neoclassical prototype) or the AK type, found that the tax structure, and thus fiscal policy, matter for economic growth. Among others, I refer here only to a limited number of theoretical papers where there is some indication that a switch from direct to indirect taxation could have some positive growth rate effects: King and Rebelo (1990), Pecorino (1993), Rebelo and Stokey (1995), Milesi-Ferretti and Roubini (1998), Coleman (2000), Turnovsky (2000).


On a policy standpoint, is to be mentioned the recent short paper of Martin Feldstein (2015). Feldstein strongly argues for a fiscal policy focused on revenue neutral fiscal incentives, enacted by the individual Eurozone countries, to end the Euro crisis. Feldstein, which seems more concerned on the demand side effects of a given tax shift, writes:

«an individual Eurozone country could commit to raise its value added tax rate by two percentage points a year for the next five years with the extra revenue returned in the form of lower income tax rates. The prospect of future increases in the value added tax would stimulate consumers to spend before prices rise and would also raise the rate of consumer price inflation».

The rerunning-history simulations I present here show that the tax structure shock determines a transitory demand effect (accompanied by a modest pressure on the inflation rate) but a permanent effect on the ‘equilibrium’ growth rate.
3. Rerunning history: modeling the tax shift

Given the theoretical reference model (closed economy), I do not take into account fiscal devaluation issues\(^2\). The simulation exercise is modeled so as to test the macroeconomic effects of a revenue neutral shift from income taxes to consumption taxes. The shock occurs in 1980 and is one time-shock: it is represented by a decrease of around 1.3% of Gdp in direct taxes and a correspondent increase in indirect taxes – see Graphs 1 and 2. Therefore, even the tax ratio, defined as the fraction of direct taxes to indirect taxes, decreases. In the Merman model (Felli and Gerli, 2002), the tax revenues are endogenous and the tax rates are the implicit values obtained from these revenues and from the estimated taxable base. Then the tax ratio is endogenous too. This of course raises a problem for our exercise, that finally was settled by using a ‘policy’ tax rate in the simulations: that is, the tax rate endogenously varies with the cycle in the benchmark or baseline solution of the model but it remains constant after the shock in the alternative scenarios (of course, endogenous tax revenues are made compatible with the shocked tax rates). This solution defines what I call a ‘history compatible-equilibrium path’: it is the increase in the volume and the rate of growth of Gdp compatible with the effective history of Italian economy if only one event of this history was changed – the shift from direct to indirect taxation. After the shock, tax rates stands stationary at the new respectively lower and higher level and do not change (i.e. do not follow the observed historical distribution in the data). This is what I mean by ‘history compatible-equilibrium path’. We see this solution as the closest representation, in the context of our framework, of a dynamic equilibrium.

A final remark is worth mentioning and deals with a working definition of direct and indirect taxes. The conventional approach is to define as direct taxes those that may be adjusted to the individual features of the taxpayer and indirect taxes those that are levied on transactions irrespective of the circumstances of buyers and sellers. Thus, wage and income taxes can be classified as direct taxes and the same for most taxes on assets and wealth as long as there is a potential adjustment for individual characteristics. As far as, for example, property taxes on owner-occupied housing may be adjusted for the individual or household attributes of owners, these levies are classified as direct taxes. That is not always the case. Property taxes on motor vehicle, commercial buildings and the like, that

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\(^2\) In any case, we performed some simulations where, as tax shifting policy, social security contributions and not income taxes have been shocked. The results obtained in this case are in line, but less intense, with those obtained shocking the direct taxes.
are not easily adjustable for individual characteristics, can be considered indirect taxes, together with most taxes on transactions with differentiated rates – value added tax (VAT), sales, excises, custom tariff, etc. But, as pointed out by Atkinson (1977), there are ‘transitional’ taxes between the two categories: for example a tax like IRAP (Regional Tax on Productive Activities), conventionally classified as an indirect tax, being proportional to sales revenues, could be in principle easily adapted to individual attributes and transformed into a direct tax. This latent ambiguity in such a tax, led us to consider IRAP among the direct taxes.

The tax ratio variable has a negative estimated coefficient in the TFP equation of the model – this per se is an evidence that distortionary taxation has a negative impact on efficiency and that the tax mix matters. In other words, the empirical evidence seems to suggest that, even on a single equation basis, a disproportionate fraction of taxes impinging on the productive inputs is an obstacle to efficiency.

Graph 1 – Effective income and VAT tax rates

Graph 2 – Income and VAT tax rates difference (shocked - historical values) % points
4. Simulation results

The results obtained by introducing the tax shift into our model are presented in terms of deviations (ratios) from the control (benchmark) solution, that is the baseline simulation which tracks the historical path of Italian economy as it is replicated by our structural model (all the exogenous variables are taken unchanged over the simulation period). The disturbed simulation re-runs this history after imposing a one time-shock on the ‘policy’ tax ratio.

The main results of our exercise are summarized in the table 1 and in graphs 3, 4 and 5, where the deviation from the baseline is expressed for each variable in terms of the ratio between the ‘shocked’ and the ‘control’ estimated values.

The overall economic effects of the tax shift seem quite noteworthy both from a qualitative and quantitative standpoint. The tax shift produces a positive effect on the economy in terms of output, employment, capital stock and aggregate demand, both in level and rate of growth, together with a general improvement in fiscal balances, that is a decrease of deficit and debt, and a moderate increase of inflation.

The cumulated output effect is remarkable in terms of both the level and the growth rate. Given a ‘multiplier effect’\(^3\) of 7.5, the output level increases by 3.1% with respect to the control solution after 30 years from the shock. The dynamic behavior described by the disturbed simulation shows an irregular path during the four years after the tax shift shock – which accounts for almost two third of the long run effect – and then a continuously regular increasing profile. The output jump in the first year after the shock reaches about 4 percentage points, followed by a strong fall in the two subsequent years – Graph 3. The reason for this path can be explained by considering that in the Merman model the output depends on the endogenous TFP, which is influenced by the fiscal policy (the tax mix), \textit{coeteris paribus}\(^4\). Therefore, the strong and immediate output-response to the shock depends on the effect of the tax shift on TFP. In terms of cumulated rates of growth, the output shows a deviation of 5.2 percentage points at the end of the simulation period – see Table 1.

Since the dynamic structural econometric models do not encompass

\(^3\) The multiplier effect has been calculated as the percentage change of each variable for one percentage point decrease in the tax ratio.

\(^4\) The other variables affecting TFP that remain unchanged in this simulation, are human capital, labor market arrangements, wage bargaining rules, legal, political and education systems, core infrastructures etc. For a complete description of the model see Felli and Gerli (2002).
the notion of a dynamic equilibrium path, it is useful to use the average compound growth rate in order to compare our results with those of the theoretical model. The average compound growth rate of output is 0.1 percentage points higher than the benchmark (control) solution – a result lower but close to that of other empirical studies and to the numerical solution of the theoretical model\(^5\). This is a crucial result, since it gives empirical support to the prediction of the theoretical model that the shift from direct to indirect tax can be interpreted as a lever of economic growth.

The simulated increase in aggregate output is caused by the increasing volumes of inputs employed in the productive system, capital and labor. In particular, employment (triggered by the lower wage taxation that stimulates both supply and demand of labor) shows a high long run multiplier effect with an increase of 6.4% in the cumulated growth rates during the simulation period. The increase in terms of employees is even higher, reaching a deviation of 9.9 percentage points with respect to the baseline solution. Even more important for employment is the attainment of a higher long run ‘equilibrium’ growth rate. The impulse-response dynamics shows a strong acceleration during the first 4-5 years following the tax shift, and a quite regular increasing path during the subsequent 10 years (see graph 4). As a result, the economic system enjoys a boost of more than one million of additional jobs and a decrease of about 2 percentage points in the unemployment rate. The higher long run level and ‘equilibrium’ growth rate of employment concerns mainly the employees.

The capital accumulation, too, shows a remarkable improvement, attaining an ‘equilibrium’ growth path higher of 0.15 percentage points. The accelerator effect works in such a way to produce a regular upper trend in the aggregate capital accumulation (see graph 4).

This improvement in the supply side of the economy has a twofold effect on the demand side\(^6\). On one hand, it affects the domestic components of aggregate demand, final consumption and gross fixed investment. On the other hand, it affects net exports. In fact, the trade balance shows the following dynamics. In the short run exports are boosted by the shock. Afterwards, imports will react to the rise in aggregate demand. As a result

\(^5\) For example, Turnovsky obtains an increase of 0.3 percentage points in the rate of growth, reducing income tax rates from 28% to 20%, which requires the introduction of a consumption tax of 13% to leave the current deficit unchanged.

\(^6\) In the Merman model used in these simulations, the output is supply-determined by means of a classical production function augmented for TFP. Therefore, output and demand do not match. Inventory variations ‘solve’ the accounting equilibrium, in terms of GDP, between aggregate supply and aggregate spending.
the trade balance suffers a negative impact of about 2% in terms of GDP.

As far as the domestic components of aggregate demand are concerned, the rise in the aggregate disposable income, primarily caused by the simulated boost in employment, quantitatively affects aggregate private consumption and investment in a quite different way, which is again consistent with the outcome of the theoretical model. In fact, the simulated consumption performance is much lower than the investment one (in both levels and growth rates), implying a switching effect from consumption to saving.

The (long run) multiplier for private consumption is the lowest among the components of internal aggregate demand. The multiplier effect on investment determines an improvement in the cumulated rate of growth around 10%, with a new equilibrium growth path very close to the one observed for employment. These figures are consistent with the accumulation of capital stock produced by the disturbed simulation, once depreciation is taken into account.

The described positive growth effects produced by the tax shift occur without significant prices tensions. In the shocked simulation, the increase in the inflation rates is negligible. At the end of the simulation period, the cumulated inflation rate is between 1.1-1.3 percentage point higher.

A concluding remark concerns the effects of the tax shock on fiscal balances and public indebtedness. The consolidation of public finance is the foremost indirect result which is obtained by the combined effect of the GDP boost and of the increase in the revenues side of the government budget.

In fact, the overall increase in government revenues is noteworthy: the new equilibrium growth rate path of the government revenues – GDP ratio is higher (0.2 percentage points) with respect to the baseline simulation. In the disturbed simulation, this effect produces a cumulated increase in this ratio of 3.1 percentage points (an yearly average increase of 0.33 percentage points) at the end of the simulation period. The joint effect of the rise in output and government revenues determines an average yearly reduction of deficit and debt ratios (-1.8 and -0.2 percentage points respectively, Table 1). In terms of the cumulated effects, at the end of the simulation period, the government debt to GDP ratio shows an impressive reduction of 32 percentage points.
Table 1 – *Cumulated Results (30 years): Deviations from baseline simulation*

<table>
<thead>
<tr>
<th></th>
<th>Growth Rates</th>
<th>Average Compound Growth Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>5.2</td>
<td>0.10</td>
</tr>
<tr>
<td>Total employment</td>
<td>6.4</td>
<td>0.18</td>
</tr>
<tr>
<td>Employees</td>
<td>9.9</td>
<td>0.25</td>
</tr>
<tr>
<td>Capital stock</td>
<td>9.4</td>
<td>0.15</td>
</tr>
<tr>
<td>Aggregate demand</td>
<td>1.6</td>
<td>0.03</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.4</td>
<td>0.03</td>
</tr>
<tr>
<td>Investment</td>
<td>9.6</td>
<td>0.19</td>
</tr>
<tr>
<td>Consumer Price</td>
<td>0.11</td>
<td>0.001</td>
</tr>
</tbody>
</table>

*Average % points Difference in levels*

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<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Government Deficit/GDP</td>
<td>-1.8</td>
</tr>
<tr>
<td>Government Debt/GDP</td>
<td>-0.2</td>
</tr>
</tbody>
</table>
Graph 3 – Output Deviations from baseline

Graph 4 – Productive Factors - Deviations from baseline
5. Concluding remarks

Even with all its limitations and caveat, the analysis I presented here shows that the option of a fiscal policy of revenue neutral tax incentives is at least to be seriously considered among all those conceivable. If pursued, this strategy might reignite growth, maintaining at the same fiscal discipline. If this approach is even politically feasible, it is completely another story.

References


Felli E.L. et al., The Macroeconomic Effects of a Shift from Direct to Indirect Taxation: Evidence from Italy, work in progress, 2011.


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Appendix – The theoretical framework

A1. Households

The economy consists of N identical individuals, each of whom has an infinite planning horizon and possesses perfect foresight. Population remains fixed over time. We shall denote individual quantities by lower case letters, and aggregate quantities by corresponding upper case letters, so that $X = N \cdot x$. We assume that the representative agent is endowed with a unit of time that can be allocated either to leisure, $l$, or to work, $1-l$ [0 < $l$ < 1]. Each individual has utility $U$ given by:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{(c_{t}l_{t}^{\psi})^{1-\sigma}}{1-\sigma} \, dt$$

(1)

with

$$\sigma > 0, \psi > 0, \psi(1 - \sigma) < \min(1, \sigma)$$

where parameter $\psi$ measures the impact of leisure on the welfare of the private agent, parameter $\sigma$ is related to the intertemporal elasticity of substitution, $s$ say, by $s = 1/\sigma$, and the first two inequalities ensure the

---

7 The CIES utility function (1) satisfies the requirements identified by Ladron-de-Guevara, Otiguera and Santos [23] and is used also by Milesi-Ferretti and Roubini [31] in a similar exercise.
normality of consumption and leisure while the last inequality ensures the concavity of the utility function, in terms of the decreasing marginal utility of both consumption and leisure as well as the negativeness of the hessian matrix.

The instantaneous budget constraint a consumer faces is:

$$\dot{k}_t = r_t k_t (1 - \tau_k) + w_t (1 - l_t) (1 - \tau_w) - c_t (1 + \tau_c)$$  \hspace{1cm} (2)$$

$$\lambda(1 + \tau_c) = e^{-\rho t} c^{-\sigma} l^{\psi - \psi \sigma}$$  \hspace{1cm} (3)$$

where $k$ is the individual's capital stock, assumed to be infinitely durable. Households derive their income by renting entrepreneurs their capital stock and by supplying labor $1 - l$ to firms in the production sector, taking the interest rate $r$ and the wage $w$ as given. Both the incomes and consumption are taxed. The capital income tax rate, labor income tax rate and consumption tax rate are $\tau_k$, $\tau_w$ and $\tau_c$, respectively. For the sake of a simpler notation, in the following we omit the subscript $t$.

The shadow value of wealth is denoted by $\lambda$. Optimization implies:

$$\lambda(1 + \tau_c) = e^{-\rho t} c^{-\sigma} l^{\psi - \psi \sigma}$$  \hspace{1cm} (4)$$

$$\lambda w(1 - \tau_w) = e^{-\rho t} c^{1-\sigma} l^{\psi - \psi \sigma - 1}$$  \hspace{1cm} (5)$$

So the contemporaneous substitutability between $c$ and $l$ is

$$c = \frac{w(1 - \tau_w) l}{\psi (1 + \tau_c)}$$  \hspace{1cm} (6)$$

In optimum $l$, hence the dynamic optimum implies:

$$-\rho - \sigma \frac{\dot{c}}{c} = \frac{\dot{\lambda}}{\lambda} = -r(1 - \tau_k)$$
or
\[
\frac{\dot{c}}{c} = \frac{r(1 - \tau k) - \rho}{\sigma} \tag{7}
\]

We also have the transversality condition:
\[
\lim_{t \to \infty} \lambda_k \exp(-\rho t) = 0 \tag{8}
\]

A2. Firms

Output of the individual firm, \( y \), is determined by the Cobb-Douglas production function:
\[
y = \alpha' G^\beta (1 - l)^\phi k^{1-\beta} \equiv \alpha' \left( \frac{G}{k} \right)^\beta (1 - l)^\phi k \tag{9}
\]
\[0 \leq \beta \leq 1, 0 < \phi < 1, \phi \leq \beta\]

where \( G \) denotes the flow of services from government as in Barro [4] and Turnovsky [41]. We assume that these services are not subject to congestion so that \( G \) is a pure public good\(^8\).

The individual firm faces positive, but diminishing, marginal physical products in all factors, non-increasing returns to scale in the private factors, capital and labor, but constant returns to scale in private and in government production expenditure. We shall assume that government claims a fraction, \( g \), of aggregate output, \( Y \), for its purchases, in accordance with:

\(^8\) In the production function (9) public services (e.g. bureaucratic services, infrastructural services, property rights, etc.) are complementary with the private inputs so that a raise in \( G \) increases the marginal productivities of both \( K \) and \( L \).
Equation (10) represents the 'size' of the government. Its determinants will be analyzed in detail the next subsection. Thus combining (9) with (10), and \( Y = Ny \), aggregate output in the economy is given by:

\[
Y = (\alpha g^\beta)^{1-\beta} (1-l)^{\frac{\phi}{1-\beta}} K, \quad \alpha = \alpha'N^\beta
\]  

(11)

and is proportional to the aggregate capital stock, i.e.,

\[
\frac{Y}{K} = (\alpha g^\beta)^{1-\beta} (1-l)^{\frac{\phi}{1-\beta}}
\]  

(12)

thereby leading to an equilibrium ongoing, endogenously determined, growth. Thus the aggregate production function is an AK technology, in which the productivity of capital depends positively upon the fraction of time devoted to work and the share of productive government expenditure. We shall assume further that labor productivity is diminishing in the aggregate, leading to the additional constraint, \( \phi < 1-\beta \).

Profit maximization leads to the equilibrium wage rate and return to capital satisfying the marginal product conditions:

\[
w = \frac{\partial y}{\partial (1-l)} = \phi \frac{y}{1-l}
\]  

(13)

\[
r = \frac{\partial y}{\partial k} = (1-\beta) \frac{y}{k}
\]  

(14)

A3. Government

We rule out a market for government bonds and assume that the government runs a balanced budget at every stage of time. The revenues from income taxes and consumption taxes are used to finance the government expenditure. The government budget constraint is:
Using (6), (10), (13) and (14) the government budget constraint becomes

\[ G = rK_\tau_k + wN(1 - l)\tau_w + C\tau_c \]  \hspace{1cm} (15)

\[ g = (1 - \beta)\tau_k + \phi\tau_w + \tau_c \frac{c}{y} \]
\[ = (1 - \beta)\tau_k + \phi\tau_w + \tau_c \frac{\phi(1 - \tau_w)l}{\psi(1 + \tau_c)(1 - l)} \]  \hspace{1cm} (16)

**A4. Market Equilibrium**

The social resource constraint is

\[ Y = C + \dot{K} + G \]  \hspace{1cm} (17)

Substituting (10) for G in (17), after some rearrangement, we obtain the dynamic for aggregate capital stock:

\[ \frac{\dot{K}}{K} = \left(1 - g - \frac{C}{Y}\right) \frac{Y}{K} \] \hspace{1cm} (18)

To ensure a positive growth rate in capital stock we should have

\[ \frac{C}{Y} < 1 - g \]

Note that by (7) and (14) we get the dynamic for aggregate consumption:

\[ \frac{\dot{C}}{C} = \frac{(1 - \beta)\frac{Y}{K}(1 - \tau_k) - \rho}{\sigma} \]  \hspace{1cm} (19)

A competitive equilibrium for the economy outlined above can be defined as follows.
Definition 1. Given the initial $K_0$, an equilibrium for the economy consists of a sequence of allocations such that:

i) households maximize their utility solving problem (1);
ii) firms maximize their profits and conditions (13) and (14) hold;
iii) government budget (16) holds.

We can now state the following.

Proposition 1. If the economy follows a balanced growth path (henceforth BGP) variables grow at a constant rate, and in particular employment is constant at a value. Along this path, rate of growth of capital and consumption, $\gamma$, is then given by:

$$\gamma = \frac{(1 - \beta)(1 - \tau_k) (\alpha g^{\beta})^{\frac{1}{1-\beta}} (1 - \bar{l})^{\frac{\phi}{1-\beta}} - \rho}{\sigma}$$  \hfill (19)

Along the BGP, the dynamics of consumption and capital rely only on labor supply:

$$\frac{\dot{C}}{C} - \frac{\dot{K}}{K} = \left(1 - \frac{\phi}{1 - \beta}\right) \frac{i}{1 - l}$$  \hfill (19)

Proof. By using (13) and totally differentiating (6) we get:

$$\frac{\dot{c}}{c} = \frac{\dot{y}}{y} + \frac{i}{l(1 - l)}$$

From this, we deduce that along a BGP, the rates of growth of $c$ and $y$ will be the same. Since aggregate and per capita variables growth the same rate, given a constant $N$, the growth rate of aggregate $C$ and $Y$ will also be the same in the BGP. Therefore, the ratio of consumption to output $C$ will be $Y$ constant in the BGP. Totally differentiating (12), we get:

$$\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} + \frac{\phi}{1 - \beta} \cdot \frac{i}{1 - l}$$
Hence in the BGP consumption, capital and output all grow at the same rate. Substituting (12) into (19) we obtain the BGP growth rate $\gamma$. Subtracting (29) from (28) we obtain:

$$\frac{\dot{C}}{C} - \frac{\dot{K}}{K} = \left(\frac{1}{l} - \frac{\phi}{1-\beta}\right) \frac{i}{1-l}$$

where $\frac{1}{l} - \frac{\phi}{1-\beta} > 0$, since $0<l<1$ and $\phi<1-\beta$. \(\frac{1}{2}\).

Combining (20) and (21), we can deduce the dynamic of leisure as follows:

$$i = \left(\frac{\dot{C}}{C} - \frac{\dot{K}}{K}\right) (1-l) = \frac{1}{l} \frac{\phi}{1-\beta} \left(1 - \frac{(1-\beta)\frac{Y}{K} (1-\tau_k) - \rho}{\sigma} - \left(1 - \frac{g - C}{Y}\right) \frac{Y}{K}\right) = B(l)$$

where

$$A(l) \equiv \frac{1}{l} - \frac{\phi}{1-\beta}$$

and

$$B(l) \equiv \left[\frac{(1-\beta)(1-\tau_k)(\alpha g^\beta)^\frac{1}{\beta} (1-l)^\frac{\phi}{1-\beta} - \rho}{\sigma} + \left(1 - \frac{\phi(1-\tau_w)l}{\psi(1+\tau_c)(1-l)}\right) (\alpha g^\beta)^\frac{1}{\beta} (1-l)^\frac{\phi}{1-\beta}\right] (1-l)$$

Since $A(l)$ is always strictly positive for all values of $l$, the equation (22)
is defined for all values of $l$ between 0 and 1. Along the BGP $l$ is constant, so the numerator $B(l)$ will be zero, i.e., where is the BGP level of leisure. To study the dynamic nature of the BGP leisure we have to sign off, calculated at the fixed point, implicitly defined by $B(l) = 0$. If this derivative is positive, the fixed point is a ‘repeller’ and the BGP is locally determinate, in the sense that if $l$ were close to but not exactly equal to then $l$ would diverge further from. Thus, the BGP with is a (locally) unique equilibrium path and we can say that there is no (local) indeterminacy in this case. If instead is negative, then is an ‘attractor’, that is if $l$ is near it will eventually approach it. So there is local indeterminacy, i.e. a continuum of equilibrium trajectories all converging to the fixed point (see Pelloni and Waldmann [36]). We have:

$$\frac{dl(l)}{dl} = \frac{B'(l)}{A(l)} - \frac{A'(l)B(l)}{A^2(l)} = \frac{B'(l)}{A(l)}$$

(since $B(l) = 0$).

We can now state the following.

Proposition 2. If a BGP equilibrium defined by $B(l) = 0$ exists, to ensure its local determinacy we should have:

$$\frac{1}{1-\beta} \left[ 1 - g - \frac{\phi(1-\tau_w)l}{\psi(1+\tau_c)(1-l)} - \frac{(1-\beta)(1-\tau_k)}{\sigma} \right] + \frac{(1-\tau_w)}{\psi(1+\tau_c)(1-l)} > 0$$

And if it is local determinate, it is also unique, so there is no transitional dynamics to it.

Proof. We have:

$$B'(l) = \frac{Y}{K} \left[ \left(1 + \frac{\phi}{1-\beta}\right) \left(1 - g - \frac{\phi (1-\tau_w) l}{\psi (1+\tau_c)(1-l)} - \frac{(1-\beta)(1-\tau_k)}{\sigma} \right) \right] + \frac{\phi (1-\tau_w)}{\psi (1+\tau_c)(1-l)} + \frac{\rho}{\sigma}$$

\text{\textasteriskcentered}
Appendix – The theoretical framework

Notice that along the BGP, since aggregate consumption and aggregate capital grow at the same rate, by (18) and (19) we have

\[
\frac{\rho}{\sigma} = \left( \frac{1 - \beta}{\sigma} - (1 - g) + \frac{C}{Y} \right) \frac{Y}{K} = \left( \frac{(1 - \beta)(1 - \tau_k)}{\sigma} - (1 - g) + \frac{\phi(1 - \tau_w)i}{\psi(1 + \tau_c)(1 - \bar{i})} \right) \frac{Y}{K}
\]

so:

\[
\frac{(1 - \beta)(1 - \tau_k)}{\sigma} - (1 - g) + \frac{\phi(1 - \tau_w)i}{\psi(1 + \tau_c)(1 - \bar{i})} > 0
\]

Substituting this for \(\rho/\sigma\) in (*) we get:

\[
B'(\bar{i}) = \frac{Y}{K} \left[ \frac{\phi}{1 - \beta} \left( 1 - g - \frac{\phi(1 - \tau_w)i}{\psi(1 + \tau_c)(1 - \bar{i})} - \frac{(1 - \beta)(1 - \tau_k)}{\sigma} \right) + \frac{\phi(1 - \tau_w)}{\psi(1 + \tau_c)(1 - \bar{i})} \right]
\]

To ensure that this condition is positive we need:

\[
\frac{1}{1 - \beta} \left( 1 - g - \frac{\phi(1 - \tau_w)i}{\psi(1 + \tau_c)(1 - \bar{i})} - \frac{(1 - \beta)(1 - \tau_k)}{\sigma} \right) + \frac{(1 - \tau_w)}{\psi(1 + \tau_c)(1 - \bar{i})} > 0
\]

So if \(\frac{1}{1 - \beta}\) is always positive, we can deduce that if BGP exists and is local determinate it is unique as from the phase diagram of (22) since we can easily see that there is no way for \(B(\bar{i})/A(\bar{i})\), which is a continuous function, to cross the horizontal axis from below two times in a row.
A5. Comparative statics

In this section our analysis focuses on the effects on labor supply and growth rate of the following fiscal experiments. Accordingly, some key results of the literature on taxation and growth will be presented as a way of introducing our contribution later on. Particularly, we will tackle the following exercises of comparative statics:

i) A ceteris paribus increase in any of the tax rate $\tau_k$, $\tau_w$, and $\tau_c$.

ii) A ceteris paribus compensatory switch in distortionary taxation through an increase in $\tau_c$ fully compensated by a simultaneous reduction in $\tau_w$.

A6. Effects of an increase in distortionary taxation

Notice that an increase in $\tau_k$, $\tau_w$ and $\tau_c$ produce the same macroeconomic effects. This result is standard in the endogenous growth literature (see Turnovsky [41]-[42]) and can be summarized by the following two propositions.

Proposition 3. The equilibrium labor supply effect of an increase in any one of the taxes is negative, i.e.

$$\text{with } i = w; c; k.$$ 

Proof. Equilibrium leisure can be expressed as the solution to $B() = 0$. The effect of our tax program on leisure can be deduced by using the total derivative of $B() = 0$ with respect to leisure and the tax given the other taxes unchanged. We then have:

$${d\hat{\ell} \over d\tau_k} = - {\partial B(\hat{L}) \over \partial \tau_k} / B'(\hat{L}) =$$

$$= {1 - \beta \left(1 - \hat{\ell}\right) \over \sigma \phi \left[\frac{1}{1-\beta} \left(1 - g - \frac{\phi(1-\tau_w)\hat{I}}{\psi(1+\tau_c)(1-\hat{I})} - \frac{(1-\beta)(1-\tau_k)}{\sigma}\right) + \frac{(1-\tau_w)}{\psi(1+\tau_c)(1-\hat{I})}\right]} < 0$$
Appendix – The theoretical framework

Since the denominators of these equations are all positive, so as their numerators, the equilibrium leisure will increase if there is an increase in any one of these taxes, which means that the balanced labor supply will decrease if any one of the taxes is raised. Comparing the last two equations, we can see that the dampening effect on equilibrium labor supply of $\tau_c$ is smaller than that of $\tau_w$, with the former just being $(1-\tau_w)/(1-\tau_c)$ proportional to the latter. $\frac{1}{2}$

Proposition 4. The equilibrium growth effect of an increase in any one of the taxes is negative.

with $i = w; c; k$.

Proof. The growth effect of tax $\tau_k$ can be derived from (20) as:

$$\frac{d\tilde{y}}{d\tau_k} = \frac{\partial \gamma}{\partial \tau_k} + \frac{\partial \gamma}{\partial \tilde{y}} \frac{d\tilde{y}}{d\tau_k} =$$

$$\frac{1 - \beta}{\sigma} \left[ 1 + \frac{1 - \tau_k}{\sigma} \left( 1 - g - \frac{\phi(1-\tau_w)\tilde{y}}{\psi(1+\tau_c)(1-\tilde{y})} - \frac{(1-\beta)(1-\tau_k)}{\sigma} \right) + \frac{(1-\tau_w)}{\psi(1+\tau_c)(1-\tilde{y})} \right] < 0$$

The growth effect of tax $\tau_c$ is:
We can derive the growth effect of tax $\tau_w$ as:

$$\frac{d\gamma}{d\tau_w} = \frac{\partial \gamma}{\partial \bar{l}} \frac{d\bar{l}}{d\tau_w} =$$

$$- \frac{Y}{K(1-l)} \cdot \frac{\phi(1-\tau_k)(1-\tau_w)\bar{l}}{\sigma \psi(1+\tau_c)(1+\tau_c)^2 \left[ \frac{1}{1-\beta} \left( 1 - g - \frac{\phi(1-\tau_0)\bar{l}}{\psi(1+\tau_0)(1-l)} - \frac{(1-\beta)(1-\tau_k)}{\sigma} \right) + \frac{(1-\tau_w)}{\psi(1+\tau_c)(1-l)} \right]} < 0$$

We can derive the growth effect of tax $\tau_c$ as:

$$\frac{d\gamma}{d\tau_c} = \frac{\partial \gamma}{\partial \bar{l}} \frac{d\bar{l}}{d\tau_c} =$$

$$- \frac{Y}{K(1-l)} \cdot \frac{\phi(1-\tau_k)\bar{l}}{\sigma \psi(1+\tau_c)(1+\tau_c)^2 \left[ \frac{1}{1-\beta} \left( 1 - g - \frac{\phi(1-\tau_0)\bar{l}}{\psi(1+\tau_0)(1-l)} - \frac{(1-\beta)(1-\tau_k)}{\sigma} \right) + \frac{(1-\tau_w)}{\psi(1+\tau_c)(1-l)} \right]} < 0$$

Comparing the last two equations, we can find that the dampening effect on balanced growth of tax $\tau_c$ is only a proportion of $(1-\tau_w)/(1-\tau_c)$ of that of tax $\tau_w$, therefore the growth reducing effect of $\tau_c$ is smaller than that of $\tau_w$. 

The implications from the level and growth effects of the taxes is that decreasing labor income taxes as well as increasing consumption tax will improve equilibrium labor supply and growth rate, however the effect on the ratio of consumption to output is ambiguous. To analyze the tax-structure shift and its growth effect we assume that $g$ is fixed. We attempt to discuss with $g$ unchanged, the effect of any tax-structure variation on growth and welfare.

A.7 Effects of a tax shift between $\tau_w$ and $\tau_c$

In the present experiment, we assume that the fiscal authority reduces income tax and replaces it with an increase in consumption tax such that capital taxation $\tau_k$ and government budget remain unchanged. From the government budget constraint (16) and given $g$ and $\tau_k$ unchanged, the tax-structure switch follows the following rule:
Appendix – The theoretical framework

\[ dg = F'_{\tau_w} \cdot d\tau_w + F'_{\tau_c} \cdot d\tau_c = 0 \]

where we derive from (16) with respect to \( \tau_w \) to get

\[ F'_{\tau_w} \equiv \frac{dg}{d\tau_w} = \phi - \frac{\phi}{\psi} \frac{\tau_c}{1 + \tau_c} \frac{\tilde{l}}{1 - \tilde{l}} + \frac{\phi}{\psi} \frac{\tau_c}{1 + \tau_c} \frac{1 - \tau_w}{(1 - \tilde{l})^2} d\tau_w \] (23)

and with respect to \( \tau_c \), to get

\[ F'_{\tau_c} \equiv \frac{dg}{d\tau_c} = \frac{\phi}{\psi} \frac{1 - \tau_w}{1 + \tau_c} \frac{1}{1 - \tilde{l}} \left( \frac{\tilde{l}}{1 + \tau_c} + \frac{\tau_c d\tilde{l}}{1 - \tilde{l} d\tau_c} \right) \] (24)

While we can immediately see that , we need some manipulations to get the sign of . First we compute , then we plug it into (23) we deduce\(^9\):

\[ F'_{\tau_w} = \phi + \frac{\phi}{\psi} \frac{\tau_c}{1 + \tau_c} \frac{1}{1 - \tilde{l}} \left( \frac{1 - \tau_w}{1 - \tilde{l}} d\tilde{l} - \tilde{l} \right) \]

\[ = \phi \left[ 1 + \frac{\tau_c \tilde{l} \left( \frac{(1 - \beta) (1 - \tau_k)}{\sigma} - 1 + g + \frac{\phi (1 - \tau_w) \tilde{l}}{\psi (1 + \tau_c) (1 - \tilde{l})} \right)}{(1 - \beta) \psi (1 + \tau_c) (1 - \tilde{l}) \Theta} \right] \] (25)

with

\[ \Theta \equiv \frac{1}{1 - \beta} \left( 1 - g - \frac{\phi (1 - \tau_w) \tilde{l}}{\psi (1 + \tau_c) (1 - \tilde{l})} - \frac{(1 - \beta) (1 - \tau_k)}{\sigma} \right) + \]

\[ + \frac{(1 - \tau_w)}{\psi (1 + \tau_c) (1 - \tilde{l})} = \frac{B' \left( \tilde{l} \right)}{\phi Y/K} > 0 \]

\(^9\)See equation the proof of proposition 3 for the details of the calculation.
The proof for Proposition 2 in the appendix ensures that $\Theta>0$. Therefore, the term in square bracket in (25) is positive. So, even the sign of is positive. The accommodation between $\tau_w$ and $\tau_c$ in a revenue-neutral tax-structure shift experiment is then:

$$\frac{d\tau_w}{d\tau_c} \bigg|_{\bar{g}, \bar{r}_k} = -\frac{F'_c}{F'_w} < 0$$ (26)

which means that a unit decrease in labor income tax rate should be compensated by/unit increase in consumption tax rate to keep government size in this model unchanged.

The following proposition summarizes the previous results and states that a revenue neutral switch between income tax and consumption tax is good for long run growth of output per capita.

Proposition 5. Along the BGP, a revenue-neutral switch in distortionary taxation through a reduction in $\tau_w$ accompanied by a simultaneous compensatory hike in $\tau_c$, keeping capital taxation unchanged, implies:

$$\text{sign} \left( \frac{d\gamma}{d\tau_c} \bigg|_{\bar{g}, \bar{r}_k, \tau_w \text{ adjusts}} \right) = \text{sign} \left( \frac{\bar{l}}{\psi (1 - \bar{l})} - 1 \right)$$ (27)

i.e. it will increase equilibrium labor supply and growth rate iff

Proof. Notice that

$$\frac{d\gamma}{d\tau_c} \bigg|_{\bar{g}, \bar{r}_k, \tau_w \text{ adjusts}} = \frac{\partial \gamma}{\partial \bar{l}} \frac{d\bar{l}}{d\tau_w} \bigg|_{\bar{g}, \bar{r}_k, \tau_w \text{ adjusts}} \frac{d\tau_w}{d\tau_c} \bigg|_{\bar{g}, \bar{r}_k, \tau_w \text{ adjusts}} + \frac{\partial \gamma}{\partial \bar{l}} \frac{d\bar{l}}{d\tau_c}$$

where the first item on the RHS is positive while the second item is negative. Using proof pf Proposition 3 and 4 and (26) we get:
\[
\frac{\partial \gamma}{\partial \tilde{l}} \frac{d \tilde{I}}{d \tau_w} \bigg|_{\tilde{g}, \tilde{r}_w, \tau_c \text{ adjusts}} = \\
\frac{Y \phi}{K} \frac{(1 - \tau_c)(1 - \tau_w) \tilde{l}}{\psi(1 + \tau_c)} \frac{1}{2} \left( \frac{1 - \tau_c}{1 + \tau_c} \left( 1 - g + \frac{\phi(1 - \tau_w) \tilde{l}}{\psi(1 + \tau_c)(1 - \tilde{l})} \right) \right)
\]
Notice that the sign of this equation is due to the sign of \( \phi \). We can prove that this term is positive, given the transversality condition (8) and the non-increasing return to scale of the production function \( (\phi \leq \beta) \). Transversality condition (8) requires that the growth rate is less than the net interest rate, i.e., \( \gamma < r(1 - \tau_k) \). Using \( \gamma \) we can express \( \gamma \) equal to as in (18). Using (14) for \( r \) we have net interest rate equal to \( (1-\beta)(1-\tau_k) \) Y. Therefore we establish

\[
1 - g - \frac{C}{Y} < (1 - \beta)(1 - \tau_k)
\]

in which we substitute (16) for \( g \) to obtain:

\[
\frac{\bar{l}}{\psi (1 - \bar{l})} > \frac{\beta - \phi \tau_w}{\phi (1 - \tau_w)} = 1 + \frac{\beta - \phi}{\phi (1 - \tau_w)}
\]

Since \( \phi \leq \beta \) we can easily find that

\[
\frac{\bar{l}}{\psi (1 - \bar{l})} > 1
\]

which is in fact a necessary condition for the economy to avoid an explosive growth path. This condition holds iff. This completes the proof.

In this economy, therefore, a revenue-neutral tax switch can permanently affect the labor supply, thereby raising capital productivity and stationary growth rate. The fall in income tax brings about a raise of the return on labor, inducing people to work more. This effect is partially offset by the higher tax on consumption, which induce a switch in favor of leisure. The dominant effect depends crucially upon parameter \( \psi \), which measures how much leisure affects individuals’ welfare.