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JACOPO MARIA RICCI

STABILITY OF ENTROPIC RISK MEASURES

RomaTrE-Press
2024

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Dipartimento di Economia Aziendale

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Stability of Entropic Risk Measures
Jacopo Maria Ricci

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Stability of Entropic Risk Measures

Jacopo Maria Ricci*

ABSTRACT

The estimation error is a crucial issue which affects portfolio selection. The majority of the studies concern the most famous risk measures such as variance, Mean Absolute Deviation (MAD), and Conditional Value-at-Risk (CVaR). On the other hand, to date, there seems to be no study concerning the stability of entropic risk measures such as EntropicValue-at-Risk (EVaR) and Relativistic Value-at-Risk (RLVaR).
Using both simulated and historical data, we found that, while EVaR and CVaR exhibit a similar stability profile, RLVaR is much more sensitive to noise.

KEYWORDS: Portfolio Selection; Noise sensitivity; Estimation error; Entropy.

ABSTRACT

L'errore di stima è un tema cruciale che riguarda la selezione di portafoglio. La maggior parte degli studi concerne le misure di rischio più famose, quali la varianza, il Mean Absolute Deviation (MAD), e il Conditional Value-at-Risk (CVaR). D'altro canto, al momento, non sembrano esserci studi riguardanti la stabilità di misure di rischio entropiche quali l'Entropic Value-at-Risk (EVaR) e il Relativistic Value-at-Risk (RLVaR).
Utilizzando dati sia simulati sia storici, abbiamo trovato che, mentre l'EVaR e il CVaR manifestano un profilo di stabilità simile, il RLVaR è molto più sensibile al rumore.

PAROLE CHIAVE: Selezione di portafoglio; Sensibilità al rumore; Errore di stima; Entropia.

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* Roma Tre University - Department of Business Studies.
Via S. D'Amico 77, 00145 Rome, Italy; jacopomaria.ricci@uniroma3.it.

1. Introduction

The problem of quantifying risk has always been a fundamental issue in economics and finance. Indeed, it concerns several fields from risk management to asset allocation. As for the latter, starting from the seminal papers by Markowitz (1952, 1959), there have been many works dedicated to the problem of selecting a portfolio by minimizing a risk measure. Usually, the considerations concerning such risk measures only involve their mathemathical properties and the performances of the models based on such measures. However, an often overlooked issue regards the stability of the solution with respect to changes in input data. Financial portfolios tend to be composed of many assets, while, on the other hand, the time series of the prices (or of the returns) are limited. This means that the problem of estimation error is not negligible. Several authors have investigated the stability of the Mean-Variance (MV) model, i.e., Jorion (1985); Best and Grauer (1991); Broadie (1993); Britten-Jones (1999); Chopra and Ziemba (2013). To fix the issue of errors in the estimates of means and covariances of the assets, various approaches have been proposed, including restrictions on the weights (see Haugen (1997)), or the use of Bayesian shrinkage estimators instead of sample estimators (see e.g., Jorion (1985)) Other risk measures have been used in portfolio optimization since the work by Markowitz, such as MAD (Konno and Yamazaki (1991)), or CVaR (Rockafellar et al (2000)). Concerning the former, Simaan (1997) studied the estimation risk af the MAD portfolios compared to the MV ones. Kaut et al (2007); Goldberg et al (2013); Caccioli et al (2018) analyzed the estimation error of the CVaR model. Kondor et al (2007) carried out an extensive empirical analysis concerning four famous minimum risk models, i.e., the models that minimize Variance, MAD, CVaR and MaxLoss. Finally, Cesarone et al (2020) examined the sensitivity of several minimum risk, maximum risk-gain ratio and risk diversification portfolios. In the recent years, entropy has progressively gained more importance in finance, and, specifically, in portfolio selection. Two important risk measures based on entropy have been developed, i.e., the Entropic Value-at-Risk (Ahmadi-Javid (2012); Ahmadi-Javid and Fallah-Tafti (2019)) and the Relativistic Value-at-risk (Cajas (2023)). In this work, we want to study how these two new entropy-based risk measures are affected by noise, compared to some classical risk measures. We focus on long-only portfolios, and on minimum risk models, since the estimation of the expected return poses an additional issue and further amplifies the estimation error (see e.g., Jorion (1985); Best and Grauer (1991); Chopra and Ziemba (2013); DeMiguel et al (2009)). This work is structured as follows. Section 2 is dedicated to the description of the models analyzed. In Section 3, we provide the methods we use to perturb the data. Then, in Section 4, we describe the stability measures used to evaluate the stability, and the data sets. Then, we analyze the results. Finally, in Section 5, we draw some conclusions.

2. Portfolio selection models

In this section, we describe the models for which we study the sensitivity w.r.t. changes in input data. Let us first introduce some notation. Let p_{it} denote the price of asset i at time t, and $r_{it} = \frac{p_{it} - p_{i(t-1)}}{p_{i(t-1)}}$ denote its linear return, with $i = 1, \ldots, n$ and $t = 1, \ldots, T$. Additionally, let $x = \{x_1, \ldots, x_n\}$ be the vector of portfolio weights, so that $R_t(x) = \sum_{i=1}^{n} r_{it} x_i$ denotes the return of the portfolio at time t. Finally, $\mu = \{\mu_1, \ldots, \mu_n\}$ is the vector of the means of the assets, and Σ is the covariance matrix, whose entries σ_{ij} are the covariances of the returns between asset i and asset j. Below, we report all the portfolio selections models considered in this work. These are all minimization models whose general formulation is the following:

$$\begin{cases} \min\limits_{x} & Risk(x) \\ \text{s.t.} & \\ & \sum\limits_{i=1}^{n} x_i = 1 \\ & x_i \geq 0 \qquad i = 1, \ldots, n \end{cases}$$

Here we only describe the risk measures, however, to find the optimization problems, we refer the reader to the references cited. The risk measure used in this work are Variance, CVaR, EVaR and RLVaR. The portfolio variance is expressed as:

$$\sigma_P^2(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j$$

The portfolio selection model that minimizes the variance is that by Markowitz (1952). The CVaR, also called *expected shortfall*, is a downside risk measure which became very popular in the recent years. Given a confidence level ε, the $CVaR_\varepsilon$ is defined as the expected value of the losses $l_P(x)$ in the worst $100\varepsilon\%$ cases, where $l_P(x) = -R_P(x) = -\sum_{i=1}^{n} R_i x_i$. Note that R_i is the random variable which represents the return of the i^{th} asset, with r_{it} being its realization at time t.

Mathematically, the $CVaR_\varepsilon$ is expressed as follows:

$$\mathrm{CVaR}_\varepsilon(x) = \inf_{t\in\mathbb{R}}\left\{t + \frac{1}{\varepsilon}E[l_P(x) - t]_+\right\}, \quad \varepsilon \in [0, 1].$$

The CVaR is minimized via the linear programming (LP) problem by Rockafellar et al (2000). As in Kondor et al (2007), we set $\varepsilon = 0.30$ (for the following risk measures, as well). The EVaR is a new risk measure proposed by Ahmadi-Javid (2012); Ahmadi-Javid and Fallah-Tafti (2019), and it is defined as

$$EVaR_\varepsilon(x) := \inf_{z>0}\left\{\frac{1}{z}\ln\left(\frac{M_{l_P(x)}(z)}{\varepsilon}\right)\right\},$$

where $M_X(z) = E[e^{zX}]$ denotes the moment generating function of a random variable X. The EVaR represents the smallest upper bound of VaR stemming from the Chernoff inequality (see Chernoff (1952)). Given ε, the EVaR is an upper bound both to the VaR, and to the CVaR. The minimization of the EVaR of the portfolio was first considered by Ahmadi-Javid and Fallah-Tafti (2019), and it is a convex program whose constraints and variables are independent of the length of the series T. However, in our experiments, we consider the reformulation by Cajas (2021), which is a convex programming problem efficiently solvable by several softwares. The last measure considered is the RLVaR (Cajas (2023)), a measure whose dual representation is the following:

$$RLVaR^k_\varepsilon(x) = \sup_{Z\in M_{\varphi,\beta}} E[Z' l_P(x)],$$

where $M_{\varphi,\beta} = \{Z \geq 0, E[Z] = 1, E[Z\ln_{\{k\}}(Z)] \leq \ln_{\{k\}}(\frac{1}{\varepsilon T})\}$, $\ln_{\{k\}}(x) = \frac{x^k - x^{-k}}{2k}$ is the k-logarithm function (see (Kaniadakis, 2001)) and $k \in (0, 1)$ denotes the deformation parameter. Regarding k, we have that, for a given level ε, $\lim_{k\to 0} RLVaR^k_\varepsilon(x) \approx EVaR_\varepsilon(x)$, and, $\lim_{k\to 1} RLVaR^k_\varepsilon(x) \approx \operatorname{ess\,sup}(x)$. In this work, we set $k = 0.30$. Additionally, for a fixed ε, the following inequalities hold:

$$EVaR_\varepsilon(x) \leq RLVaR^k_\varepsilon(x) \leq \operatorname{ess\,sup}(x)$$

The optimization problem consisting in the minimization of RLVaR can be found in Cajas (2023).

3. Methods for perturbating the inputs

Here we describe the procedure used to evaluate the stability of the portfolios x^* around the true portfolio $x^{(0)}$. First, we find the "true" optimal portfolio for all the models, for a given original returns matrix, which we assume to be true. Then, for a fixed level $\frac{n}{T}$, we perturb the original data by generating $M = 50$ new samples, either with Monte Carlo or with bootstrap methods. The samples generated are statistically equivalent to the original data. Therefore, let $R = \{R_1, R_2, \ldots, R_n\}$ be the multivariate returns. Then, to perturb the returns we use the following methods:

1. Monte Carlo method:

   i standard normal market, i.e., $R \sim N(0, I)$, where I is the identity matrix.

   ii Normal market, with $R \sim N(\mu, \Sigma)$, where μ and Σ are estimated from the real-world datasets described in section 4.1. We use the returns from july 2020 to may 2024 ($\tau = 1000$ days) to compute the sample mean vector and sample covariance matrix, μ and Σ, respectively. Therefore, for a fixed n, we generate M samples with dimensions $\tau \times n$, where τ changes according to the value of $\frac{n}{T}$ considered (see Section 4).

2. Resampling method (see e.g. Michaud and Michaud (2007)): for each data set consisting in a $\tau \times n$ returns matrix, we generate the new samples via bootstrapping, i.e., redrawing the historical returns with replacement. As for the Normal market instance, M $\tau \times n$ samples are generated. Here we consider two block bootstrap sizes (BBS), where BBS= $\{1, 3\}$ with replacement.

4. Empirical Analysis

4.1. Data sets

Here we list the data sets we used in this work. All the data sets consist in daily returns computed from daily prices, adjusted for dividends and stock splits, obtained from Thomson Reuters Datastream. The data sets are the following:
All the problems have been implemented in Matlab 23.2 on a workstation with Intel(R) Xeon(R) CPU E5-2623 v4 (2.6 GHz, 64 Gb RAM) under MS Windows 10 Pro.

Table 1: List of the daily datasets analyzed.

Index	Abbreviation	Country	# Assets	From - To
Dow Jones Industrial Average	DJIA	USA	27	March 2018–May 2024
Euro Stoxx 50	STX50	EU	47	March 2018–May 2024
FTSE 100	FTSE	UK	78	March 2018–May 2024
NASDAQ-100	NDX	USA	67	March 2018–May 2024

4.2. Stability measures

In order to test the stability of the "perturbed" optimal portfolio x^* w.r.t. the "true" optimal portfolio $x^{(0)}$, as in Cesarone et al (2020), we consider the three following measures:

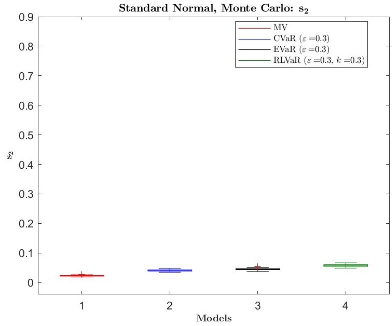
1. $s_2 = ||x^{(0)} - x^*||_2$, i.e., the Euclidean norm of the difference between x^* and $x^{(0)}$;
2. $s_1 = \sum_{i=1}^{n} |x^{(0)} - x^*|$, i.e., the l_1 norm of the difference between x^* and $x^{(0)}$;
3. $s_{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x^{(0)} - x^*)^2}$, i.e., the root mean square error of the difference between x^* and $x^{(0)}$.

Below, we provide the results for both of the methods described in Section 3. For reasons of space, and since the metrics analyzed provide similar outcomes, we only report the results concerning s_2. The results regarding the remaining measures can be found in the supplemental file (upon request) `StabilitySupplemental.xlsx` (see description in `Readme.txt`).

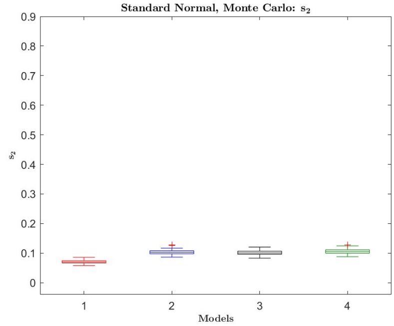
4.3. Monte Carlo method

4.3.1. Standard normal market

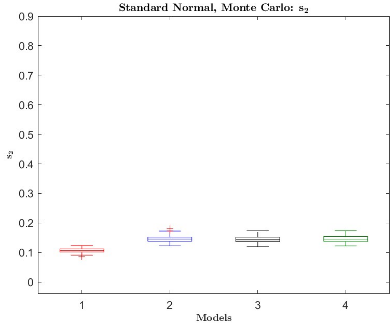
Here we consider the instance where the returns are distributed according to a multivariate standard normal distribution, i.e., $R \sim N(0, I)$. Figures from 1(a) to 1(d) show the boxplots of the distance s_2 for $\frac{n}{T} = \{0.05, 0.4, 0.95, 1.5\}$, while Table 2 displays expected value (Mean) and interquantile range (IQR) of the same measure for $\frac{n}{T} = \{0.05, 0.4, 0.7, 0.95, 1.5\}$. In order to rank the results in the tables, we assign different colors to the performances of the models analyzed.



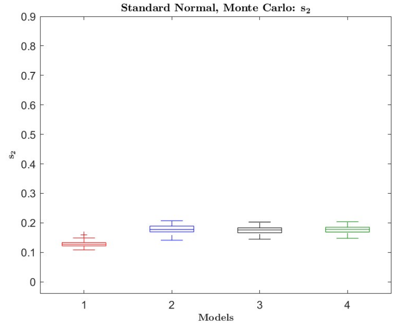
(a) Boxplot for $\frac{n}{T} = 0.05$



(b) Boxplot for $\frac{n}{T} = 0.4$



(c) Boxplot for $\frac{n}{T} = 0.95$



(d) Boxplot for $\frac{n}{T} = 1.5$

Figure 1: Dispersion of the optimal portfolios around the true" portfolio in the standard normal market

Specifically, for each column, colors range from deep-green (best) to deep-red (worst). As it can be noticed, the boxplots of s_2 move up and widen as the $\frac{n}{T}$ ratio increases, meaning that both Mean and IQR increase with this ratio. The minimum Variance portfolio is the most stable among all. The EVaR model seems to be the second best, with a few exceptions regarding especially the IQR for higher $\frac{n}{T}$ values. RLVaR is nearly always the worst model in terms of stability. However, as the $\frac{n}{T}$ ratio increases, CVaR tends to be less stable than RLVaR, and, for $\frac{n}{T} = 1.5$ it is the model with the worst performance in terms of stability.

	$\frac{n}{T}$									
	0.05		**0.4**		**0.7**		**0.95**		**1.5**	
	Mean	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**
MV	0.023	0.002	0.070	0.008	0.092	0.008	0.107	0.010	0.128	0.011
CVaR	0.041	0.004	0.103	0.011	0.132	0.013	0.147	0.014	0.178	0.020
EVaR	0.045	0.004	0.101	0.010	0.128	0.013	0.144	0.015	0.176	0.017
RLVaR	0.058	0.006	0.106	0.011	0.131	0.015	0.145	0.017	0.178	0.016

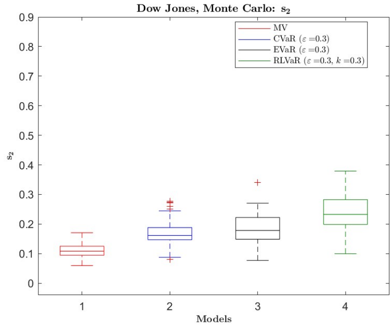
Table 2: Statistics of the distance of the optimal portfolios from the "true" optimal portfolio for the standard normal market

4.3.2. Normal market

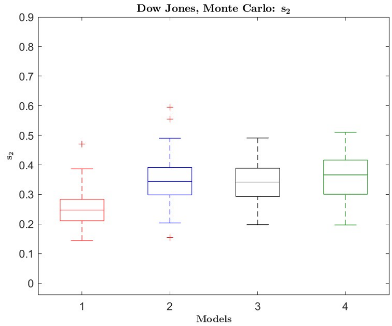
Here we discuss the results concerning the returns following a multivariate normal distribution, i.e., $R \sim N(\mu, \Sigma)$, where the mean μ and the covariance matrix Σ are estimated from the real-world datasets. Here the MV model proves, again, to be nearly always the least dispersed model, with the only exceptions being for interquantile range of the Dow Jones and NASDAQ-100 data sets, for $\frac{n}{T} = \{0.7, 0.95, 1.5\}$ (see Tables from 3 to 6). As also shown in Figures from 2(a) to 4(d), CVaR is usually more stable than EVaR for lower $\frac{n}{T}$, while the opposite tends to be true for higher values of the ratio. The RLVaR portfolio seems to yield the least accurate solution, with a few exceptions concerning only IQR.

4.4. Resampling method

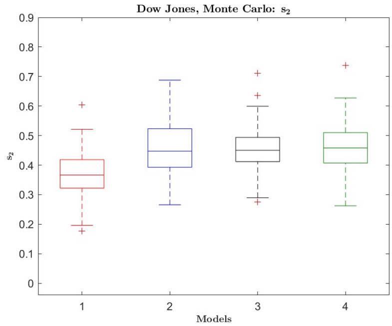
We provide here the results regarding the resampling method applied to the historical returns. Since the results for both BBS are very similar, we report here only the results where we take BBS=3. The analysis concerning the case where BBS=1 can be found in the supplemental file (upon request) `StabilitySupplemental.xlsx` (see description in `Readme.txt`). As it is possible to notice in Figure from 5(a) to 7(d) and in Tables from 7 to 10, the MV model always attains the best performance in terms of stability, while the RLVaR model is always the worst performing. For this method, the differences in the performances of these two models are much more pronounced. This is testified by the fact that the average of s_2 of MV is usually half than that of RLVaR, and, in some cases, it is even less than that. As



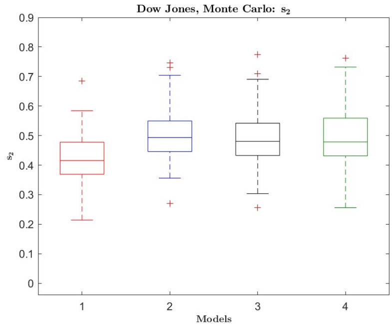
(a) Boxplot for $\frac{n}{T} = 0.05$



(b) Boxplot for $\frac{n}{T} = 0.4$

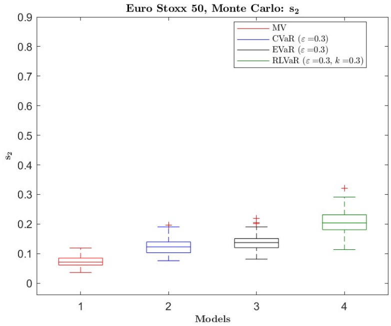


(c) Boxplot for $\frac{n}{T} = 0.95$

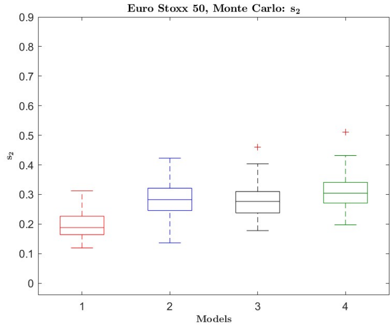


(d) Boxplot for $\frac{n}{T} = 1.5$

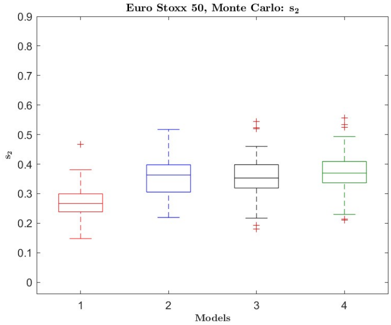
Figure 2: Monte Carlo method: dispersion of the optimal portfolios around the “true” portfolio in the Dow Jones data set



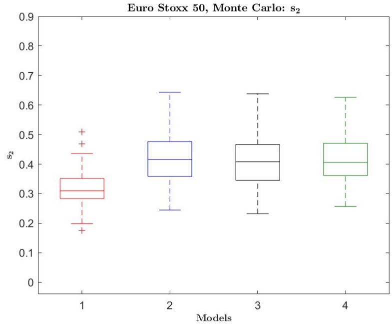
(a) Boxplot for $\frac{n}{T} = 0.05$



(b) Boxplot for $\frac{n}{T} = 0.4$

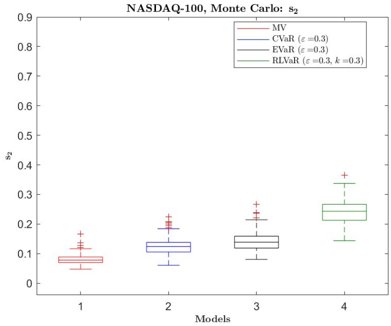


(c) Boxplot for $\frac{n}{T} = 0.95$

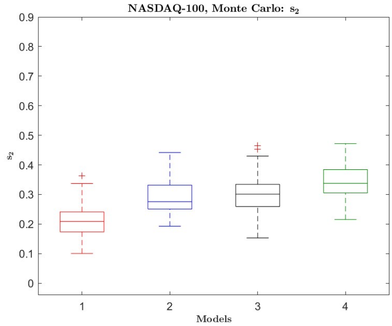


(d) Boxplot for $\frac{n}{T} = 1.5$

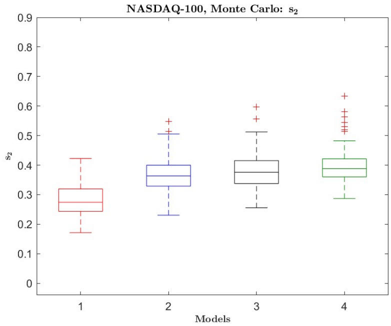
Figure 3: Monte Carlo method: dispersion of the optimal portfolios around the "true" portfolio in the Euro Stoxx 50 data set



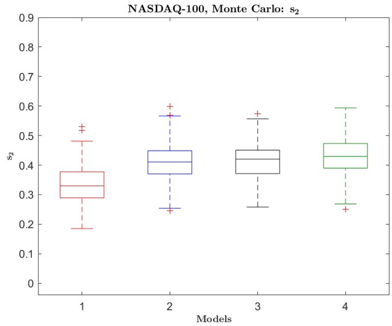
(a) Boxplot for $\frac{n}{T} = 0.05$



(b) Boxplot for $\frac{n}{T} = 0.4$



(c) Boxplot for $\frac{n}{T} = 0.95$



(d) Boxplot for $\frac{n}{T} = 1.5$

Figure 4: Monte Carlo method: dispersion of the optimal portfolios around the "true" portfolio in the NASDAQ-100 data set

	$\frac{n}{T}$									
	0.05		**0.4**		**0.7**		**0.95**		**1.5**	
	Mean	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**
MV	0.110	0.031	0.250	0.072	0.316	0.081	0.367	0.097	0.416	0.109
CVaR	0.168	0.041	0.344	0.093	0.405	0.101	0.458	0.131	0.505	0.104
EVaR	0.183	0.073	0.340	0.095	0.412	0.112	0.452	0.082	0.493	0.109
RLVaR	0.238	0.084	0.360	0.116	0.421	0.118	0.460	0.103	0.492	0.128

Table 3: Monte Carlo method: mean and IQR of the distance from the “true” optimal portfolio for the Dow Jones dataset

	$\frac{n}{T}$									
	0.05		**0.4**		**0.7**		**0.95**		**1.5**	
	Mean	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**
MV	0.074	0.023	0.194	0.063	0.238	0.065	0.270	0.061	0.316	0.068
CVaR	0.124	0.036	0.282	0.075	0.333	0.094	0.358	0.092	0.416	0.118
EVaR	0.136	0.031	0.281	0.072	0.328	0.085	0.357	0.079	0.408	0.121
RLVaR	0.204	0.050	0.305	0.070	0.346	0.086	0.375	0.072	0.418	0.109

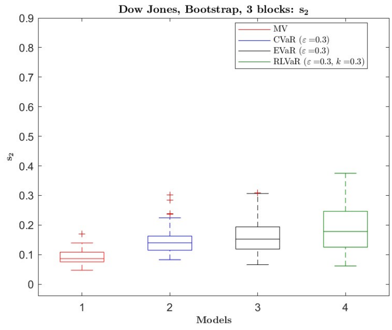
Table 4: Monte Carlo method: mean and IQR of the distance from the “true” optimal portfolio for the Euro Stoxx 50 dataset

	$\frac{n}{T}$									
	0.05		**0.4**		**0.7**		**0.95**		**1.5**	
	Mean	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**
MV	0.063	0.016	0.176	0.043	0.203	0.046	0.232	0.047	0.273	0.053
CVaR	0.097	0.025	0.232	0.043	0.278	0.052	0.305	0.060	0.357	0.064
EVaR	0.118	0.029	0.243	0.055	0.279	0.048	0.311	0.056	0.356	0.061
RLVaR	0.199	0.042	0.281	0.057	0.306	0.059	0.333	0.067	0.368	0.063

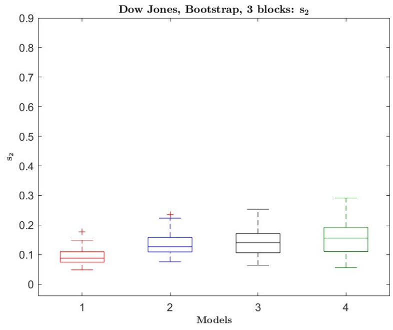
Table 5: Monte Carlo method: mean and IQR of the distance from the "true" optimal portfolio for the FTSE 100 dataset

	$\frac{n}{T}$									
	0.05		**0.4**		**0.7**		**0.95**		**1.5**	
	Mean	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**
MV	0.082	0.019	0.209	0.068	0.258	0.076	0.281	0.076	0.336	0.088
CVaR	0.127	0.033	0.288	0.081	0.338	0.082	0.368	0.071	0.413	0.079
EVaR	0.144	0.040	0.299	0.075	0.344	0.070	0.378	0.078	0.417	0.079
RLVaR	0.242	0.054	0.340	0.079	0.372	0.049	0.399	0.061	0.432	0.083

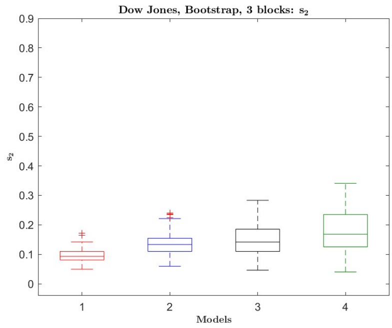
Table 6: Monte Carlo method: mean and IQR of the distance from the "true" optimal portfolio for the NASDAQ-100 dataset



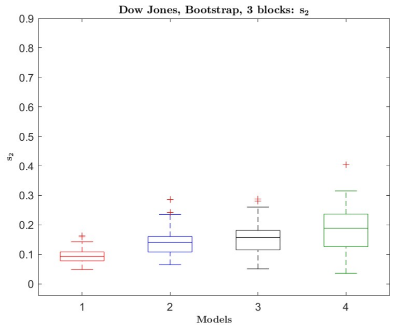
(a) Boxplot for $\frac{n}{T} = 0.05$



(b) Boxplot for $\frac{n}{T} = 0.4$



(c) Boxplot for $\frac{n}{T} = 0.95$



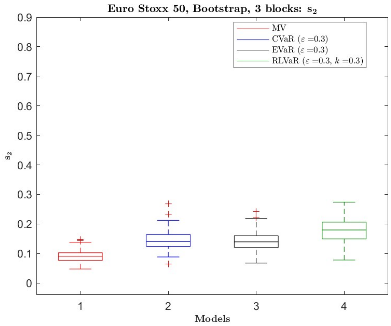
(d) Boxplot for $\frac{n}{T} = 1.5$

Figure 5: Resampling method: dispersion of the optimal portfolios around the "true" portfolio in the Dow Jones data set

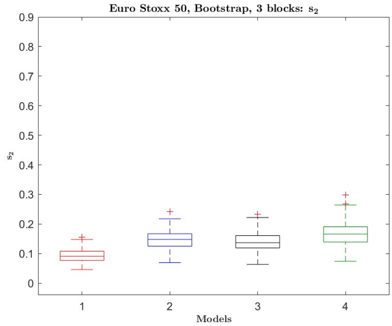
for CVaR, it is nearly always the second best, with the exception of the Eurostoxx 50 dataset, where the EVaR portfolio is less disperse than the CVaR portfolio. It is noteworthy that, for this method, unlike the Monte Carlo one, Mean and IQR don't seem to be influenced by the value of $\frac{n}{T}$.

5. Conclusions

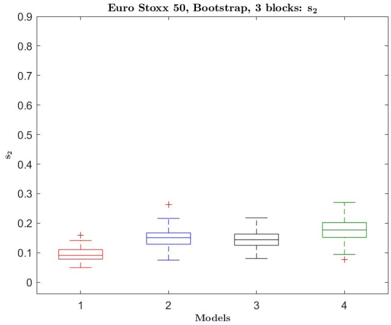
In this paper, we studied the sensitivity with respect to changes in input data of several minimum risk models. We applied different methods i.e., Monte Carlo and resampling, to long only portfolios, and we considered different stability measures. Our finding are in line with those by Kondor et al (2007) and Cesarone et al (2020).



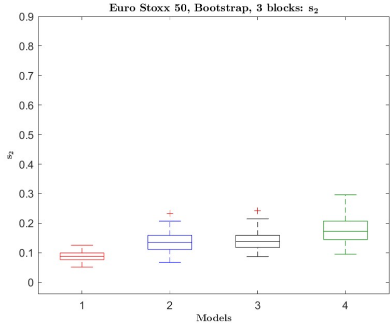
(a) Boxplot for $\frac{n}{T} = 0.05$



(b) Boxplot for $\frac{n}{T} = 0.4$

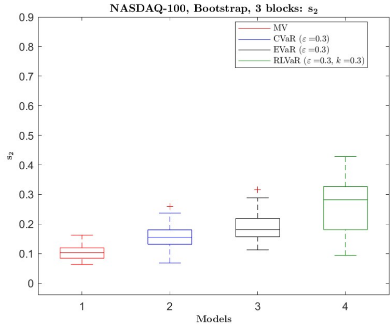


(c) Boxplot for $\frac{n}{T} = 0.95$

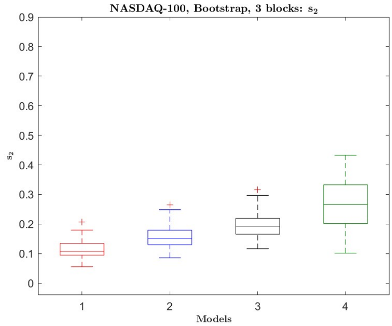


(d) Boxplot for $\frac{n}{T} = 1.5$

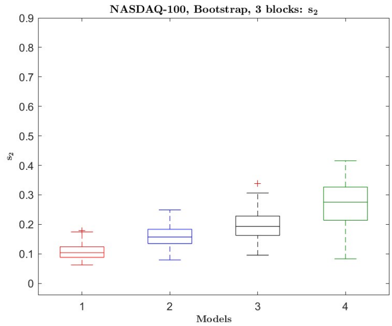
Figure 6: Resampling method: dispersion of the optimal portfolios around the “true” portfolio in the Euro Stoxx 50 data set



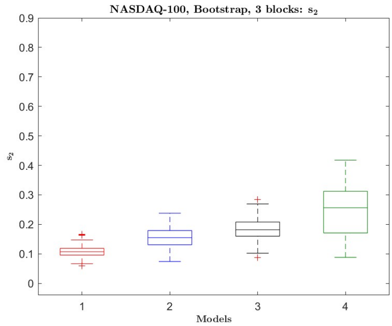
(a) Boxplot for $\frac{n}{T} = 0.05$



(b) Boxplot for $\frac{n}{T} = 0.4$



(c) Boxplot for $\frac{n}{T} = 0.95$



(d) Boxplot for $\frac{n}{T} = 1.5$

Figure 7: Resampling method: dispersion of the optimal portfolios around the "true" portfolio in the NASDAQ-100 data set

	$\frac{n}{T}$									
	0.05		**0.4**		**0.7**		**0.95**		**1.5**	
	Mean	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**
MV	0.093	0.032	0.092	0.035	0.096	0.034	0.096	0.029	0.094	0.030
CVaR	0.144	0.048	0.135	0.049	0.140	0.040	0.137	0.045	0.142	0.053
EVaR	0.160	0.075	0.142	0.065	0.159	0.068	0.151	0.075	0.155	0.066
RLVaR	0.187	0.121	0.156	0.082	0.180	0.119	0.176	0.110	0.185	0.111

Table 7: Resampling method: mean and IQR of the distance from the "true" optimal portfolio for the Dow Jones dataset, with BBS=3

	$\frac{n}{T}$									
	0.05		**0.4**		**0.7**		**0.95**		**1.5**	
	Mean	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**
MV	0.091	0.026	0.093	0.031	0.094	0.032	0.094	0.032	0.089	0.023
CVaR	0.145	0.040	0.148	0.042	0.146	0.040	0.150	0.038	0.139	0.048
EVaR	0.142	0.040	0.141	0.042	0.142	0.039	0.144	0.038	0.143	0.042
RLVaR	0.176	0.057	0.168	0.052	0.171	0.059	0.178	0.050	0.176	0.062

Table 8: Resampling method: mean and IQR of the distance from the "true" optimal portfolio for the Euro Stoxx 50 dataset, with BBS=3

	$\frac{n}{T}$									
	0.05		0.4		0.7		0.95		1.5	
	Mean	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**
MV	0.090	0.023	0.089	0.023	0.090	0.023	0.090	0.021	0.090	0.025
CVaR	0.148	0.048	0.146	0.041	0.149	0.042	0.149	0.046	0.149	0.050
EVaR	0.163	0.063	0.164	0.060	0.166	0.049	0.163	0.061	0.165	0.058
RLVaR	0.225	0.134	0.228	0.117	0.232	0.106	0.226	0.104	0.223	0.102

Table 9: Resampling method: mean and IQR of the distance from the "true" optimal portfolio for the FTSE 100 dataset, with BBS=3

	$\frac{n}{T}$									
	0.05		0.4		0.7		0.95		1.5	
	Mean	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**	**Mean**	**IQR**
MV	0.104	0.035	0.114	0.040	0.105	0.032	0.108	0.036	0.109	0.023
CVaR	0.155	0.049	0.157	0.049	0.157	0.043	0.159	0.048	0.156	0.048
EVaR	0.188	0.062	0.196	0.054	0.192	0.044	0.196	0.066	0.184	0.048
RLVaR	0.262	0.145	0.267	0.131	0.265	0.135	0.271	0.113	0.244	0.141

Table 10: Resampling method: mean and IQR of the distance from the "true" optimal portfolio for the NASDAQ-100 dataset, with BBS=3

In particular, asymmetric risk measures seem to be more sensitive to estimation error than symmetric risk measures such as the variance. Our preliminaries results, show that the entropic risk measures, especially the Relativistic Value-at-Risk, are heavily influenced by noise. Concerning the Entropic Value-at-Risk, its performances depend on the method used (Monte Carlo or resampling) and on the presence of correlation structure. Indeed, in the standard normal market, where all the assets are uncorrelated, such a model is less sensitive to noise than the Conditional Value-at-Risk. Considering the Monte Carlo case where the parameters are estimated from real-world data sets, the former model tends to be slightly less disperse than the latter, especially for higher $\frac{n}{T}$ values. However, when considering the results of the resampling method, the Entropic Value-at-Risk is nearly always less stable than the Conditional Value-at-Risk. Therefore, in this instance, the method used seems to affect the results. On the other hand, the solution provided by the minimum Relativistic Value-at-Risk model is consistently the most disperse, independently on the method used. For what concerns future research, it might be directed to studying the sensitivity of Entropic Value-at-Risk to the confidence level ε, and the sensitivity of the Relativistic Value-at-Risk to ε and to the parameter k.

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The estimation error is a crucial issue which affects portfolio selection. The majority of the studies concern the most famous risk measures such as variance, Mean Absolute Deviation (MAD), and Conditional Value-at-Risk (CVaR). On the other hand, to date, there seems to be no study concerning the stability of entropic risk measures such as Entropic Value-at-Risk (EVaR) and Relativistic Value-at-Risk (RLVaR). Using both simulated and historical data, we found that, while EVaR and CVaR exhibit a similar stability profile, RLVaR is much more sensitive to noise.

JACOPO MARIA RICCI
È assegnista di ricerca presso il Dipartimento di Economia Aziendale (Università degli Studi Roma Tre).

ROMA TRE
UNIVERSITÀ DEGLI STUDI